Solutions to Problem 1

$$\lambda_i = \begin{cases} 25\left(1 - \frac{i}{i+1}\right) & \text{for } i = 0, 1, \dots, 19 \\ 0 & \text{for } i = 20, 21, \dots \end{cases}$$

$$\mu_i = \begin{cases} 15 & \text{if } i = 1 \\ 30 & \text{if } i = 2, 3, \dots \end{cases}$$

Solutions to Problem 2

a.

$$d_0 = 1 \qquad d_1 = \frac{\lambda_0}{\mu_1} = 2 \qquad d_2 = d_1\frac{\lambda_1}{\mu_2} = 2 \qquad d_3 = d_2\frac{\lambda_2}{\mu_3} = \frac{4}{3} \qquad d_4 = d_3\frac{\lambda_3}{\mu_4} = \frac{2}{3}$$

$$d_n = 0 \quad \text{for } n = 5, 6, \dots$$

$$\Rightarrow D = 7$$

$$\Rightarrow \pi_0 = \frac{d_0}{D} = \frac{1}{7} \qquad \pi_1 = \frac{d_1}{D} = \frac{2}{7} \qquad \pi_2 = \frac{d_2}{D} = \frac{2}{7} \qquad \pi_3 = \frac{d_3}{D} = \frac{4}{21} \qquad \pi_4 = \frac{d_4}{D} = \frac{2}{21}$$

$$\pi_n = 0 \quad \text{for } n = 5, 6, \dots$$

Alternatively, using the generator matrix:

$$\mathbf{G} = \begin{bmatrix} -6 & 6 & 0 & 0 & 0 \\ 3 & -9 & 6 & 0 & 0 \\ 0 & 6 & -12 & 6 & 0 \\ 0 & 0 & 9 & -15 & 6 \\ 0 & 0 & 0 & 12 & -12 \end{bmatrix}$$

$$\begin{array}{c} \pi^\top \mathbf{G} = 0 \\ \pi^\top \mathbf{1} = 1 \end{array} \quad \Rightarrow \quad \begin{aligned} -6\pi_0 + 3\pi_1 &= 0 \\ 6\pi_0 - 9\pi_1 + 6\pi_2 &= 0 \\ 6\pi_1 - 12\pi_2 + 9\pi_3 &= 0 \\ 6\pi_2 - 15\pi_3 + 12\pi_4 &= 0 \\ 6\pi_3 - 12\pi_4 &= 0 \\ \pi_0 + \pi_1 + \pi_2 + \pi_3 + \pi_4 &= 1 \end{aligned} \quad \Rightarrow \quad \begin{aligned} \pi_1 &= 2\pi_0 \\ \pi_2 &= 2\pi_0 \\ \pi_3 &= \frac{4}{3}\pi_0 \\ \pi_4 &= \frac{2}{3}\pi_0 \end{aligned}$$

$$\Rightarrow \pi_0 = \frac{1}{7} \qquad \pi_1 = \frac{2}{7} \qquad \pi_2 = \frac{2}{7} \qquad \pi_3 = \frac{4}{21} \qquad \pi_4 = \frac{2}{21} \qquad \pi_n = 0 \quad \text{for } n = 5, 6, \dots$$

b.

$$\pi_4 = 0.09$$

c.

$$\ell = \sum_{n=0}^{\infty} n\pi_n = 1(0.29) + 2(0.29) + 3(0.19) + 4(0.09) = 1.8$$

d.

$$\lambda_{\text{eff}} = \sum_{i=0}^{\infty} \lambda_i \pi_i = 6(0.14) + 6(0.29) + 6(0.29) + 6(0.19) + 0(0.09) = 5.46$$

$$\Rightarrow w = \frac{\ell}{\lambda_{\text{eff}}} = \frac{1.8}{5.46} \approx 0.3297$$

Note that one could answer this question directly from the problem data: each phone call takes an average of 20 minutes, or 1/3 hour, and there is no queue.

Solutions to Problem 3

a. We want π_{10}:

$$\lambda = 90 \qquad \mu = 20 \qquad \rho = \frac{90}{5(20)} = 0.9$$

$$\pi_0 = \left[\left(\sum_{j=0}^{5} \frac{(5(0.9))^j}{j!} \right) + \frac{5^5 (0.9)^6}{5!(1-0.9)} \right]^{-1} \approx 0.0050$$

$$\pi_{10} \approx \frac{(90/20)^{10}}{5! 5^{10-5}} (0.0050) \approx 0.0450$$

b.

$$1 - \pi_0 \approx 0.995$$

c.

$$\ell_q = \frac{\pi_5 \rho}{(1-\rho)^2} = \frac{0.076(0.9)}{(1-0.9)^2} = 6.84 \text{ customers}$$

d.

$$w_q = \frac{\ell_q}{\lambda} = \frac{6.84}{90} = 0.076 \text{ hours} = 4.56 \text{ minutes}$$

Solutions to Problem 4

a. The interarrival times are exponentially distributed, as indicated by the first "M".

b. Since the queueing discipline is not specified, it takes on the default value, which is FIFO (first in first out).